

“Tolerance Analysis Estimator”

QUALITY TOOLS

Description of Tolerance Analysis Estimator:

Tolerance Analysis is commonly associated as an engineering tool to estimate cumulated variation of multiple component of a mechanical stack-ups and reflecting variation of total system. In general, tolerance analysis can be applied practically anywhere where process stages are happening sequentially with certain variation.

Tolerance Analysis, in general, has two main approaches – Worst Case and Statistical, RSS (Root Sum of Squares) – first being more “conservative” and latter being more “optimistic” and relying on individual process capability knowledge. Various different approaches between WC and RSS is available and in each case, responsible person/organization should evaluate associated risk(s) in a application before deciding which approach to use.

Tolerance Analysis is useful tool to estimate stack up variation of complex systems before design finalization. It also can help designers to estimate differences between optional systems and subsystems in order to minimize probability of nonconformance quality.

When to use the Tolerance Analysis Estimator:

Tolerance Analysis should be applied always when systems and/or subsystem output is not known. Even simplest calculation could reveal “unexpected” source(s) of variation (through sketching a loop diagram) and give better indication what areas to focus on to reduce variation.

How to use the Tolerance Analysis Estimator:

Assumptions behind calculations:

- All processes are centered (no mean shift).
- All process stages variations in stack-up are independent (no correlation).
- All tolerances are equal bilateral tolerances (symmetrical tolerances). If not, they need to be changed (Ex. +0.10 / -0.20 → +/-0.15).
- All process stages are assumed to follow Gaussian (Normal) distribution.
- A “float” between two tolerances are assumed to be Uniform –distribution and are assumed not to be process related (non-predictable).

Formula to calculate Worst Case, WC, tolerance (below) in Tolerance Analysis Estimator:

$$Tol, WC(\pm) = \sum_{i=1}^n Float_i + \sum_{j=1}^m Tol_j$$

Formula to calculate Root Sum of Squares, RSS, tolerance (below) in Tolerance Analysis Estimator:

$$Tol, RSS(\pm) = \sum_{i=1}^n Float_i + \sqrt{\left(\sum_{j=1}^m \left(\frac{Tol_j}{\sigma_j} \right)^2 \right)}$$

Formula to calculate Modified Root Sum of Squares, MRSS, tolerance (below) in Tolerance Analysis Estimator:

$$Tol, MRSS(\pm) = Factor_{MRSS} * \left[\sum_{i=1}^n Float_i + \sqrt{\left(\sum_{j=1}^m \left(\frac{Tol_j}{\sigma_j} \right)^2 \right)} \right]$$

Historically, $Factor_{MRSS}$, has been suggested to be between 1.4 and 1.8 [Ref. 1]. This value User can change and estimate impact of $Factor_{MRSS}$ to $Result_{MRSS}$.

$Float$ = Nominal or intended gap or idle between two tolerance stack – up component.

Tol_j = Tolerance of j^{th} tolerance component it stack up.

σ_j = Process σ – level of j^{th} tolerance stack – up component.

Usage:

NOTE ! ONLY “Yellow” cells should be filled.

1. Fill initial information.

- **Problem statement:** What is the problem what you are trying to solve?
- **Description of stack-up analysis:** What area the stack-up analysis is focusing on? What items are included in stack-up and what are not? What assumptions have been made?
- **Direction:** In technical application(s), one may want to indicate in which direction in a Local Coordinate system the analysis is made in (like; X, Y or Z –direction) (Optional).

Prepared by: [Name]	Reviewed by: [Name(s)]	Date: 2013-06-11
		Revision: []
		Direction: []
Problem statement:		
[Description]		
Description of stack-up analysis:		
[Description]		

2. Make a Loop diagram.

- **NOTE:** It can be hand made in a piece of paper, white board etc. as long as it describe current status as accurately as possible. (See section “Application of Tolerance Analysis Estimator”, Example 1.).
- Mark target of estimation with (X).
- Make a loop diagram through all stack-up components one by one (steps) and label them individually (Ref.: A,B,C... or 1,2,3 etc.).

Loop Diagram:
[Insert Your loop diagram drawing here]

3. Fill variable data in each rows according Your loop diagram.

NOTE: Each row can have only one value – either Float or Tol(+/-).

- Refer the loop diagram step names in variable data field (Ref.: A,B,C... or 1,2,3 etc.) for Your own reference.
- **Float:** Nominal or intended gap or idle between two tolerances.
- **Tol(±):** kth tolerance value (bilateral tolerances).
- **σ(±):** Process sigma –level estimation relative kth tolerance value.

#	Description	Ref.	Float	Tol. [s]	σδ	SS	%-Contrib.
1	Part1	A		1.0000	3.0000	0.1111	16.287%
2	Part2	B		1.0000	3.0000	0.1111	16.287%
3	Part3	C		1.5000	3.0000	0.2500	36.645%
4	Part4	D		0.5000	3.0000	0.0278	4.072%
5	Part5	E		1.0000	3.0000	0.1111	16.287%
6	Part6	F		0.8000	3.0000	0.0711	10.423%
7							
8							
9							
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12							
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18							
19							
20							

4. Fill targeted specification limits, UTL (Upper Tolerance Limit) and LSL (Lower Tolerance Limit), Z-shift(±σ) and MRSS –factor (*Unit Cost is optional*).

- **NOTE:** Assumption is that all tolerance stack-up components are at their nominal value and therefore stack-up is also it's nominal position.
- **UTL/LTL:** Tolerance Limits around nominal output value (nonconformance limits).
- **UnitCost:** Cost of nonconformance unit outside of USL and LSL –limits (*optional*).
- **Z-shift(±σ):** By altering this values, User can visualize how process shift impacts on CoPQ –values. When Z-shift(±σ) = 0 → CoPQ = CoPQ±X.Xσ

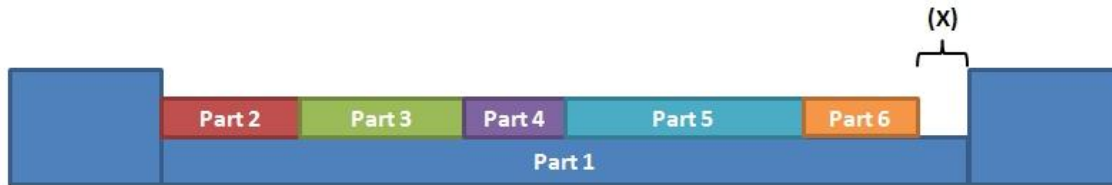
Specs:	Results:	DPM & CoPQ:
UTL 2.5000	Result(WC): ± 5.8000	DPM(>USL) 1236
LTL -2.5000	Result(RSS): ± 0.8260	DPM(<LSL) 1236
UnitCost \$1.00	Result(MRSS): ± 1.2390	Total DPM 2472
Z -shift(±δ) 1.50		CoPQ \$2,472
MRSS -factor 1.50		CoPQ±1.5δ \$126,822

Tips on use of Tolerance Analysis Estimator:

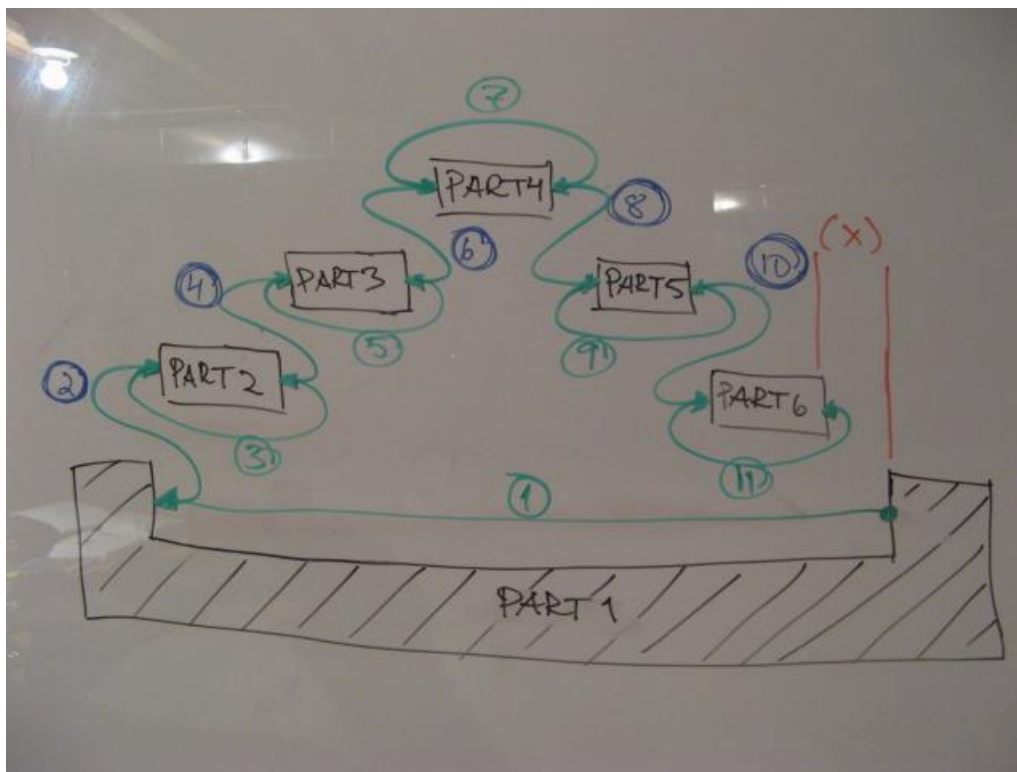
It is important to understand this tool should be used only for estimations - not an unambiguous and absolute result due to highlighted limitations and assumptions.

Application of Tolerance Analysis Estimator:

Example 1a: A loop diagram (initial stage).



Example 1b: A loop diagram (with reference labels; here #1-#11).



Example 1b: (X) = Estimation output, Items[1,3,5,7,9 and 11] are stack-up component which has process related variation. Items[2,4,6,8 and 10] are “floats” as they represent Nominal or intended gap or idle between two tolerance stack-up component if any(→ zero).

References:

Ref. 1: *Dimensioning and Tolerancing Handbook*, Paul Drake Jr., McGraw-Hill, ISBN 0-07-018131-4, 1999.

Ref. 2: *Tolerance Design – A Handbook for Developing Optimal Specifications*, C.M.Creveling, Addison Wesley Longman, Inc., ISBN 0133052346, 1997.